ALGEBRA

**List 8.**

*Symmetric matrices, bilinear and quadratic forms. Second order curves and surfaces.*

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**1.** For the matrix

$$A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix},$$

(i) find eigenvalues and orthonormal eigenbasis;

(ii) write the associated quadratic form in the standard basis and in the eigenbasis;

(iii) determine whether the matrix is positive or negative (semi-)definite.

**2.** For the matrix

$$A = \begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix},$$

(i) find eigenvalues and orthonormal eigenbasis;

(ii) write the associated quadratic form in the standard basis and in the eigenbasis;

(iii) determine whether the matrix is either positive or negative (semi-)definite.

**3.** For the quadratic form

$$Q(x, y, z) = 2x^2 + y^2 - 4xy - 4yz,$$

(i) write the matrix of the form and diagonalize it;

(ii) write the orthogonal matrix which transforms the quadratic form to the canonic form, and write the canonic form;

(iii) determine whether the quadratic form is either positive or negative (semi-)definite.

**4.** For the quadratic form

$$Q(x, y, z) = -2yz,$$

(i) write the matrix of the form and diagonalize it;

(ii) write the orthogonal matrix which transforms the quadratic form to the canonic form, and write the canonic form;

(iii) determine whether the quadratic form is either positive or negative (semi-)definite.

**5.** For the systems of vectors given below, apply the Gramm-Schmidt orthogonalization procedure to get an orthonormal basis in the subspace spanned by these vectors.

(a) $(1, 2)$, $(1, -1)$;

(b) $(2, 1), (3, 1)$;

(c) $(1, 1, 2)$, $(1, -1, 2)$;

(d) $(1, 1, 1)$, $(1, 2, 3)$;

(e) $(1, 1, 2)$, $(1, -1, 2)$, $(1, 0, 2)$;

(f) $(1, 1, 1)$, $(1, 1, -1)$, $(-1, 1, 1)$.

**6.** For the second order curve given by the equation

$$7x^2 - 2y^2 - 12xy - 10x + 6 = 0,$$

(i) write the canonical form and specify the type of the curve;

(ii) specify the coordinate system in which the curve has the canonical form;

(iii) draw the curve.

**7.** Perform the same analysis as in the above problem for the following second order curves:

(a) $34x^2 + 41y^2 + 24xy + 92x + 106y - 49 = 0$;

(b) $2x^2 + 2y^2 - 2xy + 10x - 2y - 13 = 0$;

(c) $5x^2 - 5y^2 - 12xy + 34x - 14y + 25 = 0$.

**8.** For the second order surface given by the equation

$$4x^2 + 4y^2 + z^2 + 8xy + 8xz + 4yz + 18x + 18z + 18 = 0,$$

(i) write the canonical form and specify the type of the surface;

(ii) specify the coordinate system in which the curve has the canonical form;

(iii) sketch the picture of the surface.

**9.** For the second order surface given by the equation

$$x^2 + y^2 - 2z^2 + 8xy + 4xz + 3 = 0,$$

(i) write the canonical form and specify the type of the surface;

(ii) specify the coordinate system in which the curve has the canonical form;

(iii) sketch the picture of the surface.

**10.** For the second order surface given by the equation

$$5x^2 + 7y^2 + 6z^2 + 4xy - 4yz - 18 = 0,$$

(i) write the canonical form and specify the type of the surface;

(ii) specify the coordinate system in which the curve has the canonical form;

(iii) sketch the picture of the surface.